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Mechanical Behavior of U-Shaped Bellows and Shape Optimal Design Using Multiple Objective Optimization Method

Byoung-Gab Ko*, Gyung-Jin Park and Wan-Ik Lee*****

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A bellows is frequently applied in piping systems for absorbing mechanical movement. Its geometry is an axial symmetric shell, which is composed of two toroidal shells and one annular plate. The mechanical behavior of U-shaped bellows under axial force and internal pressure is estimated by changing the dimensions of the geometric parameters. The changing ranges of the geometric dimensions is so selected as to invest the results with practical environments in many fields. The minimization of strength and spring rate is considered simultaneously as a multiple objective function. The weighting objective method is implemented, in which a vector function is transformed to a scalar function. The structure is analyzed by the energy method for toroidal sections. Optimization is carried out by the Recursive Quadratic Programming algorithm.

Key Words: Bellows Expansion Joint, Quadrant-Toroidal Shell, Multiple Objective Optimization Method, Weighting Objective Method, EJMA Standard, Corrugation

Nomenclature

a_1, a_2 : Outer and inner torus radius, respectively
C_n, C_p : Factors from EJMA
D : $Et^3/12(1-\nu^2)$, shell stiffness
$\bar{F}$: $Fa/4\pi D$, dimensionless total axial forc
h : Corrugation height
l : Length of annular plate
$\bar{p}$: $3(1-\nu^2)a^3/Et^3$, dimensionless pressure
q : Corrugation pitch
r : Arbitary circumferential radius
t : Material thickness
v, w : Meridional and normal displacement, respectively
y, z : Axial and radial displacement, respectively
λ : A/r, radius ratio
ϕ, θ : Meridional and circumferential angle, respectively
σ_t : Total stress from EJMA
μ : $3(1-\nu^2)a^4/r^2t^2$, shell parameter, dimensionless

Subscripts

ϕ, θ : Meridional and circumferential direction, respectively
1, 2 : Outer and inner torus, respectively

* Department of Graduate Student, Mechanical Design and Production Engineering, College of Engineering, Hanyang University
** Department of Mechanical Engineering, College of Engineering, Hanyang University
*** Department of Mechanical Design and Production Engineering, College of Engineering, Hanyang University 17 Haengdang-Dong, Seongdong-Ku, Seoul 133-791, Korea

1. Introduction

Bellows is commonly used in piping systems to absorb expansion and contraction in order to reduce stress in the main system. It has widespread applications including; industrial and chemical plants, fossil and nuclear power systems, heating and cooling systems, and vehicle exhaust systems. Unlike most used piping components, the bellows consists of a thin-walled shell of revolution with a corrugated meridian, in order to provide the flexibility needed to absorb mechanical movements. It is a composite shell structure consisting of at least one toroidal shell (called corrugation or convolution) and an annular plate.

Because of its geometric complex, it is difficult to analyze the behavior of bellows. The design standards used in design and manufacturing have been made based on the analysis results (Calladine, 1974 ; Kellogg, 1957 ; Kraus, 1967) until the 1970's. During this period there have been several noteworthy theoretical stress studies(Clark, 1970 ; Laupa and Weil, 1962 ; Takezono, 1971) of bellows. However, each one has inherent limitations. Only a limited number of investigations(Bhavikatti, 1979 ; Chakraverti, 1976 ; Hamada, 1973) have been reported concerning shape optimization.

The objective of this paper is to present the effect of the geometric parameters on the mechanical behavior of U-shaped bellows. The loading condition is under axial force and internal pressure. The results present design graphs that enable the designer to easily evaluate the mechanical behavior of bellows. The results are compared with the practical failure examples. The shape optimization is evaluated, which the theoretical structural analysis is processed by the energy theory for the toroidal section. The optimization is performed by the Recursive Quadratic Programming algorithm. The strength and the spring rate are simultaneously considered as a multiple objective function, in order to minimize stresses of the structure and to maximize flexible capability of the bellows. A weighting objective method is applied, in which a vector function is transformed to a scalar function. The procedure for shape optimization consider the torus radii, the length of annular plate and the thickness of the structure as design variables. Stress, fatigue life, natural frequency, and buckling load are implemented to be constraint functions. The characteristics of the bellows structure are discussed. The design scheme is combined with the optimal design processes on the geometric parameters governing the bellows behavior. The optimized solution is compared with the dimensions of existing bellows.

2. Theoretical Structural Analysis

The basic geometric parameters and the nomenclature of a U-shaped bellows are illustrated in Fig. 1. The U-shaped bellows consists of two semicircular toroidal sections connected by the annular plate. The bellows may be considered as an assemblage of an individual one half-corrugation as shown in Fig. 2.

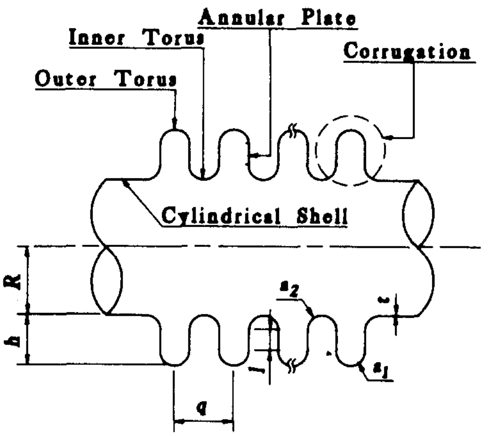


Fig. 1 Basic geometric parameters and nomenclature of bellows

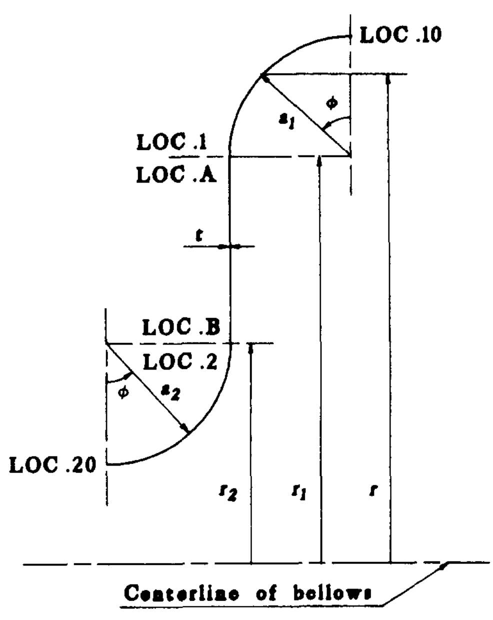


Fig. 2 Dimensions of one half-corrugation

2.1 Quadrant-toroidal section

A cross section of the quadrant-toroidal shell is shown in Fig. 3 and the deformation of a meridional element in Fig. 3(b).

The dimensionless axial and radial displacements can be expressed as ;

$$\eta=\frac{y}{a_1}=\int_0^{\phi}\beta\sin\phi d\phi$$
$$\zeta=\frac{z}{a_1}=\zeta_1-\int_{\phi}^{\pi/2}\beta\cos\phi d\phi. \qquad (1)$$

The tangent angle β is approximated by a finite trigonometric series ;

$$\beta=C_1\sin\phi+\sum_{n=1}^{k}C_{2n}\sin(2n\phi). \qquad (2)$$

The expression automatically satisfies the boundary condition that $\beta=\eta=0$ for $\phi=0$. Then, from the condition that $\beta=\beta_1$ and $\eta=\eta_1$ on $\phi=\pi/2$, it follows that

$$C_1=\beta_1$$
$$C_2=\frac{3}{2}\eta-\frac{3}{8}\pi\beta_1-\frac{3}{4}\sum_{n=2}^{k}C_{2n}$$
$$\left\{\frac{1}{(2n-1)}(-1)^{n+1}-\frac{1}{(2n+1)}(-1)^{n}\right\}. \qquad (3)$$

The bending strain energy of the shell element can be expressed as ;

$$U_b=\frac{D_1}{2a_1}\int_0^{\pi/2}\left\{(\beta')^2-2\nu\frac{a_1}{r}\beta\beta'\sin\phi+\left(\frac{a_1}{r}\right)^2\beta^2\sin^2\phi\right\}d\phi. \qquad (4)$$

The circumferential strain in the middle surface of the shell is represented by ;

$$\varepsilon_\theta=\frac{z}{r}=\frac{a_1}{r}\left(\zeta_1-\int_{\phi}^{\pi/2}\beta\cos\phi d\phi\right). \qquad (5)$$

The membrane strain energy of the shell element is ;

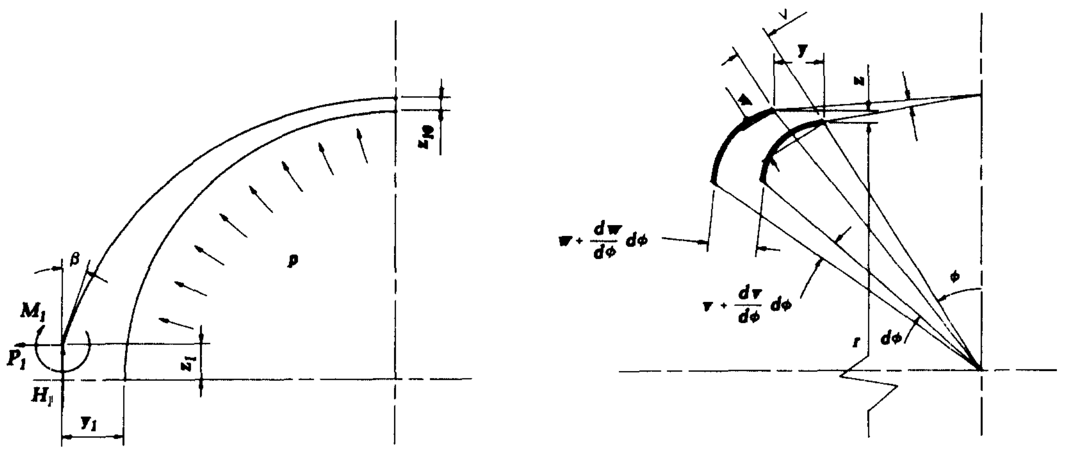
$$U_m=\frac{D_1}{2a_1}\int_0^{\pi/2}\frac{12(1-\nu^2)a_1^4}{r^2t^2}\zeta^2 d\phi. \qquad (6)$$

Confining attention initially to the outer qurdrant-toroidal shell, the work of the external forces for a unit strip can be stated in a nondimensional form as ;

$$W=M_1\beta_1+H_1a_1\zeta_1+P_1a_1\eta_1+\frac{D_1}{2a_1}\int_0^{\pi/2}8\bar{p}_1(\zeta\cos\phi+\eta\sin\phi)d\phi. \qquad (7)$$

Therefore, the total energy of the system, obtained by combining Eqs. (4), (6) and (7), is becomes

$$V=\frac{D_1}{2a_1}\int_0^{\pi/2}[(\beta')^2-2\nu\lambda_1\beta\beta'\sin\phi+(\lambda_1\beta\sin\phi)^2+4\mu_1\zeta^2-8\bar{p}_1(\zeta\cos\phi+\eta\sin\phi)]d\phi-M_1\beta_1-H_1a_1\zeta_1-P_1a_1\eta_1. \qquad (8)$$



(a) **Loading** (b) **Deformation**

Fig. 3 Loading and deformation of quadrant-toroidal section

The force-deformation relationships are determined by minimizing the total potential energy with respect to the deformations as follows ;

$$\frac{\partial V}{\partial \beta_1}=\frac{\partial V}{\partial \zeta_1}=\frac{\partial V}{\partial \eta_1}=0. \tag{9}$$

After substitution of Eqs. (1) and (2) into Eq. (8), and performing the indicated integration, which is followed by the differentiation prescribed in Eq. (9). The edge forces can be written as ;

$$\{F_i\}=[B_i]\{\delta_i\} \quad i=1,\ 2 \tag{10}$$

where,

$$\{F_i\}=\left\{\frac{a_iM_i}{D_i},\ \frac{a_i^2H_i}{D_i}-4\bar{p}_i,\ \frac{a_i^2P_i}{D_i}\right\}^T$$

$$\{\delta_i\}=\{\beta_i,\ \zeta_i,\ \eta_i\}^T.$$

The end forces and moments acting on the inner toroidal quadrant shell, which contains the crest of the corrugation, can be obtained by a derivation identical to that presented in the foregoing for the outer torus.

2.2 Annular plate

The loading and deformation occuring at annular plate is shown in Fig. 4. The edge rotations and deflections are calculated from expressions that are applicable to the symmetrical bending of annular plates. The effect of forces is neglected in the middle plane of the plate. While the edge moments M_A and M_B are arbitrary, the edge shears P_A and P_B are determined by the external loads acting on the bellows, which are comprised of the axial force F, and the end thrust of the internal pressure p.

In the notaion, the relationships between edge reactions and deformations for the annular plate can be expressed as ;

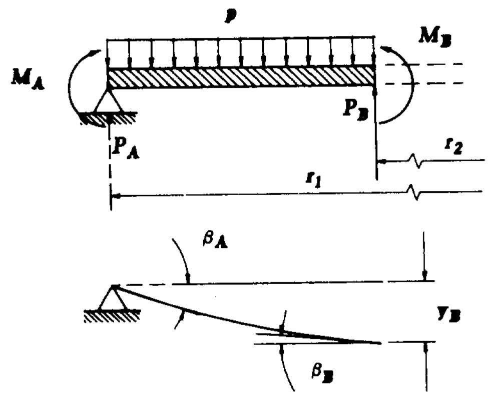
$$\{F_a\}-[c]\{\bar{F}\}=[B_a]\{\delta_a\} \tag{11}$$

where,

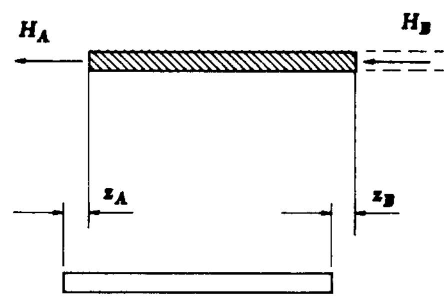
$$\{F_a\}=\frac{a_1}{D_1}\{M_A\ a_1H_A\ a_1P_A$$
$$M_B\ a_1H_B\ a_1P_B\}^T$$

$$\{\bar{F}\}=\{\bar{p}\ \bar{F}\}^T$$

$$\{\delta_a\}=\{\beta_A\ \zeta_A\ \eta_A\ \beta_B\ \zeta_B\ \eta_B\}^T.$$



(a) **Loading**



(b) **Deformation**

Fig. 4 Loading and deformation of annular plate

2.3 Compatibility and continuity

The continuity of the structure and compatibility of boundary conditions of component part are satisfied by the following equalities ;

At Location 1 : $M_1=-M_A \quad \beta_1=\beta_A$
$H_1=-H_A \quad \zeta_1=\zeta_A$
$P_1=P_A$

At Location 2 : $M_2=-M_B \quad \beta_2=\beta_B$
$H_2=-H_B \quad \zeta_2=\zeta_B$
$P_2=P_B$ (12)

Substitution of Eqs. (10) and (11) into Eq. (12) yields the following six simultaneous equations ;

$$[K]\{\delta\}=\{\bar{F}\} \tag{13}$$

where,

$$\{\delta\}=\{\beta_1\ \zeta_1\ \eta_1\ \beta_2\ \zeta_2\ \eta_2\}^T$$

and $[K]$ is the stiffness matrix of one half corrugation of bellows.

2.4 Moments and stresses

The moments and the circumferential membrane stresses are obtained from the moment-curvature relationships and Eq. (5), respectively, which follow as ;

$$M_\phi=\frac{D_1}{a_1}\{\beta_1\cos\phi+\sum_{n=1}^{k}2nC_{2n}\cos(2n\phi)$$
$$-\nu\lambda_1\sin\phi\{\beta_1\sin\phi$$
$$+\sum_{n=1}^{k}C_{2n}\sin(2n\phi)\}\}$$
$$M_\theta=\frac{D_1}{a_1}\{-\lambda_1\sin\phi\{\beta_1\sin\phi+\sum_{n=1}^{k}2nC_{2n}\sin(2n\phi)\}$$
$$+\nu\{\beta_1\cos\phi+\sum_{n=1}^{k}2nC_{2n}\cos(2n\phi)\}\}$$
$$\zeta=\zeta_1-\frac{1}{2}\{\frac{1}{2}C_1(1+\cos2\phi)$$
$$+\sum_{n=2}^{k}C_{2n}\{\frac{1}{(2n-1)}(-1)^{n+1}$$
$$-\frac{1}{(2n+1)}(-1)^{n}\}\}. \quad (14)$$

The inner toroidal section has a similar expression with β_2, D_2, a_2, λ_2. The appropriate C_{2n}-coefficients replacing the corresponding quantities having the subscript 1 in Eq. (14).

To obtain further information on the theoretical structural analysis, refer to Laupa(1962) and Lee(1991a, b).

3. Mechanical Behavior of Bellows

In order to investigate the mechanical behavior of bellows, effective stress is calculated as follows ;

$$\frac{1}{\sqrt{2}}\sqrt{(\sigma_m-\sigma_c)^2+(\sigma_c-\sigma_r)^2+(\sigma_r-\sigma_m)^2}$$
$$=\sigma_e \quad (15)$$

where : σ_e=equivalent stress

σ_m, σ_c and σ_r=meridional, circumferential and radial stresses

The stress in the direction normal to the surface is considered negligible. The two significant stress components are the meridional and circumferential, in which the formula reduces to ;

$$\sqrt{\sigma_m^2+\sigma_c^2-\sigma_m\sigma_c}=\sigma_e. \quad (16)$$

The bellows stress can be determined from Eq. (16). σ_m, σ_c are obtained from Eq. (14).

The bellows is designed and manufactured in the following ranges of geometric dimensions(Robotshaw, 1992) ;

$$0.5\le\frac{h}{q}\le1.5$$
$$\frac{h}{2R}\le0.25 \quad (17)$$

where, R represents the internal radius of bellows. The ranges proposed in Eq. (17) invest the results with practical environments within any industry.

The U-shaped bellows is an axial symmetric shell, which is a type of expansion joint commonly used in pipelines. It can withstand comparatively large deformation while bearing axial forces, as well as internal pressure loading. Thus, the bellows is examined for two loading conditions, axial force and internal pressure. The material of bellows is stainless steel, i. e., Young's modulus of elasticity E=19,300 kg/mm^2 and Poisson's ratio ν=0.3.

Since a bellows is utilized as a piping system component, the internal radius of a bellows is assumed to be the radius of a system. Therefore, the task of a designer is to select height h, pitch q and thickness t, so that bellows has a low stren-

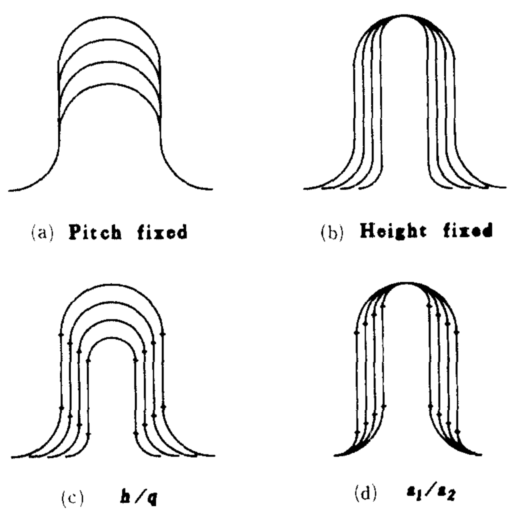
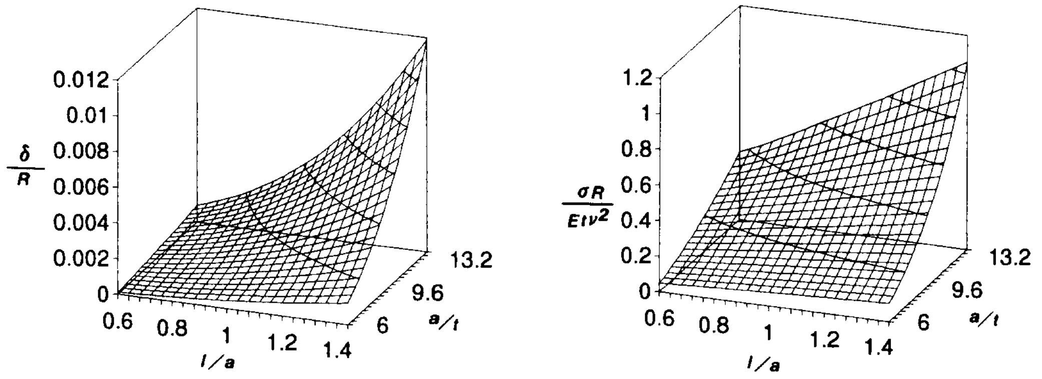


Fig. 5 Relation corrugation height(h) and pitch(q)



(a) Deflection (b) Stress

Fig. 6 Relation thickness, corrugation height and bellows behavior

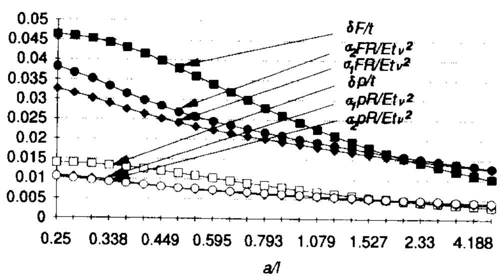


Fig. 7 Bellows behavior in case of Fig. 5(b)

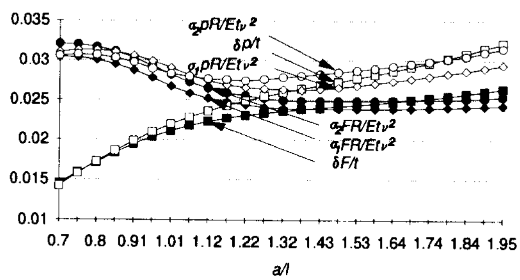


Fig. 8 Bellows behavior in case of Fig. 5(c)

gth and high flexibility. q is a function of a, while h is that of a and l, which is shown in Fig. 1. Accordingly, it is difficult to estimate the effect of the ratio h/q on the mechanical behavior of bellows. Figure 5 incorporates a case that takes into account the ratio h/q.

The curves plotted in Figs. 6 and 7 give the calculated results for Fig. 5(a) and (b), respectively. The bellows behavior corresponding to the change of t is joined in Fig. 6. From Figs. 6 and 7, as t decreases, both deflection and stress are increasing. The changes of l are presented in the same manner. Within a design condition holding either of h and q, as the ratio l/a becomes larger, the flexibility of bellows becomes better while the strength becomes worse. The bellows structure has a bending effects of a beam(annular plate), compared with a shell(toroidal shell), which influences the flexibility.

Figure 8 represents the bellows behavior for the example in Fig. 5(c). The deflection is proportional to a as shown in Fig. 8. The change in stress is either a slight decrease or increase. Within a design condition holding l, an increase of a results in an improvement of flexibility. However, the strength remains unchanged.

It is evident from Figs. 7 and 8 that the inner torus stresses are higher than the outer torus. Thus, maximum stress occurs within the inner torus. It is well known in industry that the failure of bellows mostly occurs in inner toroidal shell rather than in outer toroidal shell. The results are in accord with the practical failure examples(Lee, 1992).

Figure 9 indicates the bellows behavior for the case in Fig. 5(d), the relation a_1 and a_2.

There is no change in the deflection under axial force, while the deflection caused by internal

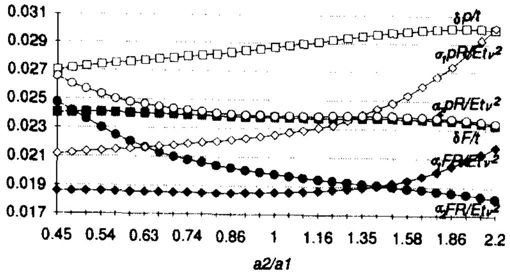


Fig. 9 Bellows behavior in case of Fig. 5(d)

pressure is slightly proportional to a_2/a_1. The tori stresses on all loading cases are inversely proportional to each a. Within a design condition holding h and q, the increase of a_2/a_1 has little effect on the flexibility under axial force. However, there's a slight improvement of the flexibility under internal pressure. As each a becomes larger, the tori strengths under all loading cases are enhanced. It is also indicated that the inner torus stresses are higher than the outer torus, considering the location where the value of a_2/a_1 is 1. In order to reduce the bellows stress, it is a good design that a_2 increases over a_1.

Hamada(1976) and Janzen(1979) derived design diagrams and design formulas based on the results from Figs. 6 and 7. The formulas are insufficient, because they do not include the results from Figs. 8 and 9. Although larger values of l and l/t are more efficient for flexibility, determination to certain design values of l and t is forced by the various conditions such as manufacturing, system environments, and etc.. Design engineers are continually faced with the problems caused by these conditions. Unfortunately, many designers do not appreciate the problems at an early design stage. Accordingly, the techniques using the shape optimal design are insisted obstinately on the bellows design. These optimization techniques provide optimal geometry of bellows within a short time. The designer is guided in modifying the initial geometry to obtain the performances, which satisfy structural behavior constraints such as stress, fatigue limit, buckling load, and natural frequency.

4. Application of Shape Optimization

4.1 Multiple objective optimization problem

A multiple objective optimization problem for mathematical programming can be formulated as follows ;

Find the vector $\bar{b}^*=[b_1^* \ b_2^* \ \cdots \ b_n^*]^T$, which satisfies the m inequality constraints ;

$$g_j(\bar{b})\geq 0 \quad j=1, 2, \cdots, m \tag{18}$$

the p equality constraints ;

$$h_j(\bar{b})=0 \quad j=1, 2, \cdots, p \tag{19}$$

and optimize the vector function ;

$$\bar{f}(\bar{b})=[f_1(\bar{b}) \ f_2(\bar{b}) \ \cdots \ f_k(\bar{b})]^T \tag{20}$$

where, $\bar{b}=[b_1 \ b_2 \ \cdots \ b_n]^T$ is the vector of decision variables defined in n-dimensional space.

This study adopts a weighting objective method(Osyczka, 1984), in which a vector function is transformed into a scalar function. The basis of this method consists in adding all the objective functions together using different weighting coefficients for each. It means that the multiple objective optimization problem changes to a scalar optimization problem by creating one function of the form ;

$$f(\bar{b})=\sum_{i=1}^{k} w_i f_i(\bar{b}) \tag{21}$$

where, $w_i\geq 0$ are the weighting coefficients representing the relative importance of the criteria. It is usually assumed that ;

$$\sum_{i=1}^{k} w_i=1. \tag{22}$$

Since the results of solving an optimization model using Eq. (21) can vary significantly as the weighting coefficients change, a necessary approach is to solve the same problem for many different values of w_i. Unfortunately, little(Lee, 1991a, b) is known on how to choose these coefficients. In order to resolute these difficulties, Eq. (21) can also be transformed into ;

$$f(\bar{b})=\sum_{i=1}^{k} w_i f_i(\bar{b}) c_i \tag{23}$$

where, c_i are constant multipliers.

The best results are usually obtained if $c_i=1/$

f_i^0, and f_i^0 denotes the minimum value of the ith function.

4.2 Mathematical formulation

The problem of finding the optimal dimensions of a bellows can be stated in the mathematical form as ;

Minimize :

$$f(\bar{b})=w_1\frac{1}{2f_1^0}\sum_{i=1}^{2}(\sigma_e^p+\sigma_e^f)_i + w_2\frac{K_{sr}}{f_2^0} \tag{24}$$

subject to

$$g_1(\bar{b})\equiv(\sigma_e^p+\sigma_e^f)_1\leq\sigma_y \tag{25}$$

$$g_2(\bar{b})\equiv(\sigma_e^p+\sigma_e^f)_2\leq\sigma_y \tag{26}$$

$$g_3(\bar{b})\equiv\left\{\frac{1308}{(\sigma_t-38)}\right\}^{3.4}\leq N_c \tag{27}$$

$$g_4(\bar{b})\equiv p_{si}\leq\frac{1.4t_p^2\sigma_y}{h^2C_p} \tag{28}$$

$$g_5(\bar{b})\equiv p_{sc}\leq\frac{0.34\pi K_{sr}}{N^3q} \tag{29}$$

$$g_6(\bar{b})\equiv f_{ax}\leq\frac{C_n}{1.5}\sqrt{\frac{K_{sr}}{W}} \tag{30}$$

$$g_7(\bar{b})\equiv f_{la}\leq\frac{C_n(2R+h)}{Nq}\sqrt{\frac{K_{sr}}{W}}. \tag{31}$$

In order to enhance the strength, t should be increased and other geometric parameters(a_1, a_2 and l) be decreased. These changes of the geometric parameters reduce the flexibility of the structure. In a problem where both axial force and internal pressure are significant, it may be difficult to get a feasible solution. Therefore, a mathematical programming approach is needed in the design of bellows, in order to ensure low strength and high flexibility. Hamada(1973) has selected maximization of deflection per unit length, and Bhavikatti(1979) has selected minimization of reaction transferred to the system as the objective function. These are the satisfactory objective functions, however the consideration for strength is deficient. This study adopts a multiple objective function that simultaneously considers strength and flexibility. The multiple objective function is composed of the minimization of strength and the maximization of flexibility as expressed in Eq. (24). The first and second term in Eq. (24) are the strength and the reciprocal of the spring rate, respectively. The stresses in Eq. (24) are given by Eqs. (14) and (16), and the spring rate K_{sr} is obtained from Eq. (13). σ_e^p and σ_e^f expresses the maximum equivalent stress caused by internal pressure and axial force. σ_y is the minimum yield stress of bellows material used in the design.

The constraints are imposed on the stresses of the structure as given in Eqs. (25) and (26). The studied constraints are induced from EJMA standard(1993), which have been adopted as the design criteria of bellows. Fatigue life and in-plane instability constraints are indicated in Eqs. (27) and (28). Column buckling is detrimental to the bellows performance in that it can greatly reduce both fatigue life and pressure capacity. Equation (29) represents the column buckling constraint, and N is the number of corrugations in the bellows. To avoid a resonant response in the bellows, the constraints on axial and lateral natural frequency are provided in Eqs. (30) and (31).

From Figs. (6)~(9), t is the most influential parameter, closely followed by l, and lastly a. It is evident that h, q, and t are three principal design variables for bellows. In the present investigation the design variable vector $\bar{b}$ consists of the following ;

$$b_1=a_1\ ;\ b_2=a_2\ ;\ b_3=l\ ;\ b_4=t.$$

A general-purpose optimization system, IDESIGN3(Arora and Tseng, 1986) is utilized for the optimal design procedure. The Recursive Quadratic Programming algorithm in IDESIGN3 is selected to solve the problem. The objective function and constraints are coded in FORTRAN language and supplied to the system. Sensitivities are obtained by the finite difference method.

5. Results and Discussions

Mostly in bellows design, either total length or diameter is limited by the system environments. Such design limitations in the practical field are usually classified as follows.

(1) Design limitation holding bellows outer diameter(corrugation height).

Table 1 Initial value and side bound for design variables

Design Variables	Initial Value	Lower Bound	Upper Bound
1	1.500	0.500	10.000
2	1.500	0.500	10.000
3	8.000	0.500	30.000
4	0.500	0.100	3.000

(2) Design limitation holding bellows length(corrugation pitch).

(3) Design limitation holding both bellows outer diameter and length(both corrugation height and pitch).

Considering the geometry of bellows, a design in case (3) determines a and t, except for l. In the optimization process the numerical values in the design limitations are constrained like the ones in the existing design. For example, the numerical value on case(1) approximately becomes 11 as noted in Table 1.

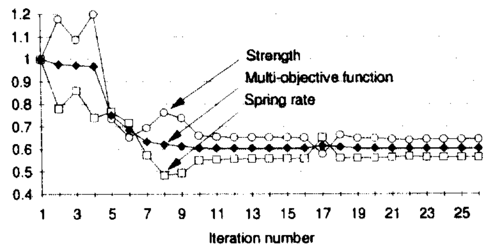
The shape optimal design process is divided into two parts. In the first part, the shape optimal design is processed without design limitations. The correlation of the geometric parameters can be systematically investigated. In the second part, the shape optimal design is processed with design limitations, in order to provide the optimal values and to compare with the existing design.

Model optimized here is bellows that is being equipped at the exhaust system. The initial values of the optimization are the ones of the bellows in automobiles.

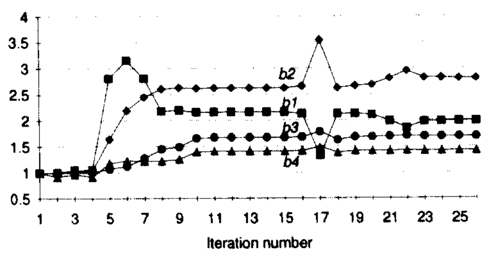
5.1 Optimal design without design limitations

Figure 10 shows the optimization processes of objective function, design variables, and constraints.

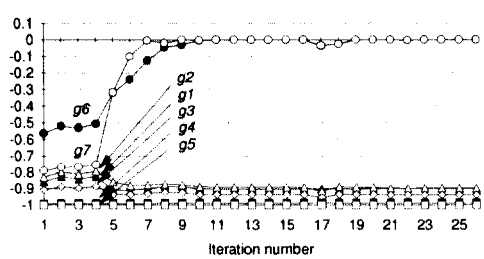
The behavior of strength and spring rate corresponding to multiple objective function is joined in Fig. 10(a). Plotted data in multiple objective function and design variables are values to initial ones, and constraint data are the normalized. The optimization process takes 26 iterations to satisfy the constraints. The strength and the spring rate respond sensitively to an imperceptible change of t, as shown in Fig. 10. As t becomes thin, the strength goes down. The spring rate goes in an inverse movement. At the 5th iteration, both strength and spring rate show an uniform decline



(a) Objective function



(b) Design variables



(c) Constraints

Fig. 10 Optimization process for multiple objective function

Table 2 Comparison with existing design and optimal design

Item	Existing Design	Optimization without Limitations	Optimization with Limitations		
			CASE 1)	CASE 2)	CASE 3)
b_1	1.500	2.984	3.193	0.788	0.688
b_2	1.500	4.207	3.294	2.462	2.562
b_3	8.000	13.510	4.763	19.560	8.000
b_4	0.500	0.701	0.399	0.788	0.488
K_{sr}	5.304	2.983	3.857	3.263	4.941
Strength	6.397	4.118	5.810	4.442	6.269

without regard to the changes of t. This type of optimization trend is accounted for by the considerable increase of a. In the optimization process of constraints, the natural frequency constraints move closer to the boundary of feasible region at the 8th iteration. In order to enhance the strength, t becomes thick. The remaining design variables approximately increase, in order to improve the flexibility.

5.2 Optimal design with design limitations

The optimal design results are listed in Table 2, including the optimal design results without the design limitations.

In case 3), the optimal design results are under the control of t, because t has a greater influence upon the bellows behavior as compared with a_2/a_1. The optimized thickness is similar to that of the existing design. The bellows behavior followed by this optimal trend have identical consequences. It is estimated that such results are due to choice of the dimensions of existing design looked upon as the design limitations. It is evident from all optimal results that a_2 is larger than a_1.

The bellows is a system component rather than a main structure in the system. Thus, there can be a several questions in the result, which are obtained in the present investigation. The optimized bellows data should be compatible with the total system or the environment of location equipped with bellows. Therefore, it is necessary to investigate the bellows as well as the mechanical behavior of the total system or the environment of location.

6. Conclusions

The results on the mechanical behavior and the shape optimal design of U-shaped bellows can be summarized as follows ;

(1) Within a design condition holding either of corrugation height(h) and pitch(q), as the ratio annular plate to torus radius(l/a) becomes larger, the flexibility of bellows becomes better while the strength becomes worse. In the bellows structure, the bending effects of a beam(annular plate) compared with those of a shell(toroidal shell) have a great influence on the improvement of flexibility.

(2) Within a design condition holding the annular plate, an increase of torus radius results in an improvement of flexibility. However, the strength remains unchanged.

(3) Within a design condition holding both corrugation height and pitch, the increase of a_2/a_1 has little effect on the flexibility under axial force. However, there's a slight improvement of the flexibility under internal pressure. As each torus radius becomes larger, the tori strengths under all loading cases are enhanced.

(4) In order to reduce the bellows stress, it is a good design that the inner torus radius increases over the outer torus.

(5) When the multiple objective function simultaneously selects both strength and spring

rate, the thickness becomes thick, in order to enhance the strength. The remaining design variables approximately increase, in order to improve the flexibility.

(6) When the optimal design is processed with design limitations in an actual field, the thickness and annular plate are marked in comparison with the existing design within a design limitation holding either of corrugation height and pitch. Within a design limitation holding both corrugation height and pitch, the optimized thickness is similar to that of the existing design. The bellows behavior followed by this optimal trend have identical consequences.

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